Pink Kangaroo

a member of the Association Kangourou sans Frontières

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Solutions

These are polished solutions and do not illustrate the process of failed ideas and rough work by which candidates may arrive at their own solutions.

It is not intended that these solutions should be thought of as the 'best' possible solutions and the ideas of readers may be equally meritorious.

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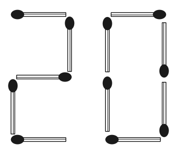
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
D	A	C	C	E	C	A	D	D	B	E	E	B	E	B	A	B	C	D	E	C	B	D	A	B

1. Carolina has a box of 30 matches. She begins to make the number 2022 using matchsticks. The diagram shows the first two digits. How many matchsticks will be left in the box when she has finished?



A 20 B 19 C 10 D 9 E 5

Solution **D**

Each of the twos in "2022" requires 5 matches, so 15 altogether. And she needs six matches for the zero, which leaves $30 - 15 - 6 = 9$ matches.

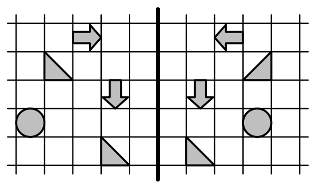
2. A square has the same perimeter as an equilateral triangle whose sides all have length 12 cm.
What is the length, in cm, of the sides of the square?

A 9 B 12 C 16 D 24 E 36

Solution **A**

The triangle has perimeter $3 \times 12 = 36$ cm. The length, in cm, of the sides of the square is $36 \div 4 = 9$ cm.

3. Some shapes are drawn on a piece of paper. The teacher folds the left-hand side of the paper over the central bold line. How many of the shapes on the left-hand side will fit exactly on top of a shape on the right-hand side?

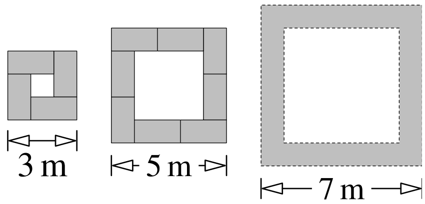


A 1 B 2 C 3 D 4 E 5

Solution **C**

A shape on the left-hand side will fit exactly over a shape on the right-hand side if it is a mirror image and the same distance away from the fold line. Therefore, the top three shapes will fit exactly, but the circles are not the same distance from the fold line and the lower triangles are not mirror images of each other.

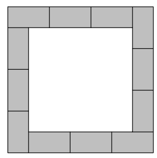
4. Katrin arranges tables measuring 2 m by 1 m according to the number of participants in a meeting. The diagrams show the plan view for a small, a medium and a large meeting.
How many tables are needed for a large meeting?



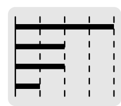
A 10 B 11 C 12 D 14 E 16

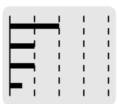
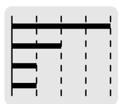
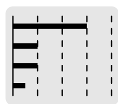
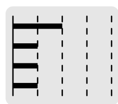
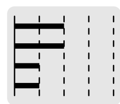
Solution **C**

Every 7 m length of the square consists of one 1 m edge of a table and three 2 m edges. Thus every side of the square uses three tables, and Katrin needs $4 \times 3 = 12$ tables altogether.



5. On Nadya's smartphone, the diagram shows how much time she spent last week on four of her apps. This week she halved the time spent on two of these apps, but spent the same amount of time as the previous week on the other two apps.
Which of the following could be the diagram for this week?



A  B  C  D  E 

Solution **E**

In Diagram E the times for the first and third apps have been halved, while the other two are unchanged. It can be easily checked that the other diagrams do not work.

6. There were five candidates in the school election. After 90% of the votes had been counted, the preliminary results were as shown on the right. How many students still had a chance of winning the election?

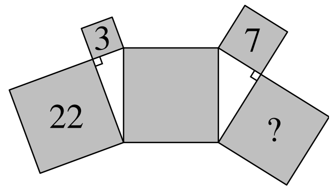
Henry	India	Jenny	Ken	Lena
14	11	10	8	2

A 1 B 2 C 3 D 4 E 5

Solution **C**

The 45 votes already cast are 90% of those available. So the remaining 10% is $45 \div 9 = 5$ votes. If Henry wins at least two of the five votes then he is certain to win the election. If India or Jenny win all five of these votes, they would be ahead of Henry. But if Ken or Lena secure five more votes, they would still be behind Henry. Hence only Henry, India and Jenny still have a chance of winning.

7. Five squares and two right-angled triangles are positioned as shown. The areas of three squares are $3\,\text{m}^2$, $7\,\text{m}^2$ and $22\,\text{m}^2$ as shown.
What is the area, in m^2, of the square with the question mark?

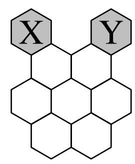


A 18 B 19 C 20 D 21 E 22

Solution **A**

Notice that the central square shares an edge with both of the right-angled triangles and in each case the shared side is the hypotenuse of the triangle. By Pythagoras' Theorem, the area of the central square is equal to the sum of the areas of the squares on the shorter sides. By considering the triangle on the left we see the area, in m^2, of the central square is $22 + 3 = 25$. Then, by considering the triangle on the right we see that the unknown area, in m^2, is $25 - 7 = 18$.

8. A ladybird aims to travel from hexagon X to hexagon Y, passing through each of the seven unshaded hexagons once and only once. She can move from one hexagon to another only through a common edge. How many different routes could she take?



A 2 B 3 C 4 D 5 E 6

Solution **D**

Any such route will need to travel anticlockwise around the outer ring of unshaded hexagons. At some point the ladybird must enter the central hexagon and then exit it to the next available outer hexagon. There are five points at which the ladybird could enter the central hexagon (since she cannot do it from the final unshaded hexagon), and each gives a different route, hence five routes.

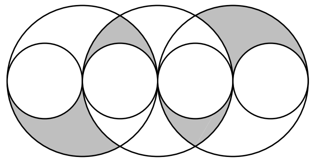
9. Adam laid 2022 tiles in a long line. Beata removed every sixth tile. Carla then removed every fifth tile. Doris then removed every fourth tile. Lastly, Eric removed all of the remaining tiles.
How many tiles did Eric remove?

A 0 B 337 C 674 D 1011 E 1348

Solution **D**

Beata leaves five-sixths of the tiles. Carla leaves four-fifths of the remaining tiles. Doris leaves three-quarters of what's left. Hence the number of tiles which Eric removes is $\frac{5}{6} \times \frac{4}{5} \times \frac{3}{4} \times 2022 = \frac{3}{6} \times 2022 = \frac{1}{2} \times 2022 = 1011$.

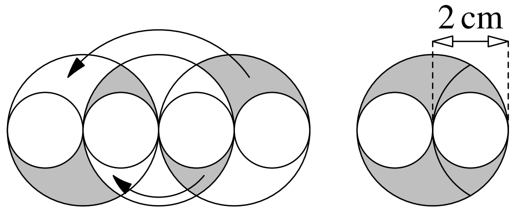
10. The centres of the seven circles shown all lie on the same line. The four smaller circles have radius 1 cm. The circles touch, as shown.
What is the total area of the shaded regions?



A π cm^2 B 2π cm^2 C 3π cm^2 D 4π cm^2 E 5π cm^2

Solution **B**

If the shaded pieces on the right-hand side are reflected in a central vertical line, the total shaded area is then the area of one large circle minus the areas of two small circles. The radius of each large circle is 2 cm so the shaded area, in cm^2, equals $\pi \times 2^2 - 2 \times \pi \times 1^2 = 4\pi - 2\pi = 2\pi$.



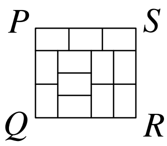
11. Gran's first grandchild guessed that Gran was 75, the second 78 and the third 81. It turned out that one of them was mistaken by 1 year, another one by 2 years and the other by 4 years. What is Gran's age?

A 76 B 77 C 78 D 79 E impossible to determine

Solution **E**

Gran could be 77 (which is 1 below 78, 2 above 75 and 4 below 81). But she could also be 79 (1 above 78, 2 below 81, 4 above 75). Hence it is impossible to determine her age from the information given.

12. Twelve congruent rectangles are placed together to make a rectangle $PQRS$ as shown. What is the ratio $PQ : QR$?



A 2 : 3 B 3 : 4 C 5 : 6 D 7 : 8 E 8 : 9

Solution **E**

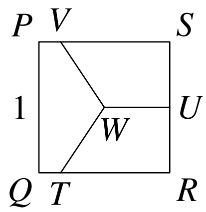
Let l be the length of the long side, and w be the length of the short side of each rectangle. Then $PS = 3l$ and $QR = 3w + l$ so $2l = 3w$ (since $PS = QR$). Then $QR = 3w + l = 3w + \frac{3}{2}w = \frac{9}{2}w$. Also, $PQ = 2l + w = 4w$. Hence the ratio $PQ : QR$ is $4 : \frac{9}{2}$, which is 8 : 9.

13. A rabbit and a hedgehog participated in a running race on a 550 m long circular track, both starting and finishing at the same point. The rabbit ran clockwise at a speed of 10 m/s and the hedgehog ran anticlockwise at a speed of 1 m/s. When they met, the rabbit continued as before, but the hedgehog turned round and ran clockwise. How many seconds after the rabbit did the hedgehog reach the finish?

Solution **B**

Initially their relative speed, in m/s, is $10 + 1 = 11$, so the time in seconds which elapses before they meet is $550 \div 11 = 50$ seconds. The hedgehog takes 50 seconds to get back again, and the rabbit covers the same distance in one-tenth of the time (5 seconds). Hence a difference of 45 seconds.

14. The diagram shows a square $PQRS$ of side-length 1. W is the centre of the square and U is the midpoint of RS. Line segments TW, UW and VW split the square into three regions of equal area. What is the length of SV?



A $\frac{1}{2}$ B $\frac{2}{3}$ C $\frac{3}{4}$ D $\frac{4}{5}$ E $\frac{5}{6}$

Solution **E**

Let the length of SV be x. Since the three areas are equal, each must be equal to one third. We know $UW = \frac{1}{2}$ and $SU = \frac{1}{2}$, so the area of the trapezium $SVWU$ is $\frac{1}{2} \times (x + \frac{1}{2}) \times \frac{1}{2} = \frac{1}{3}$. Multiplying both sides by 4, we get $(x + \frac{1}{2}) = \frac{4}{3}$ so $x = \frac{4}{3} - \frac{1}{2} = \frac{5}{6}$.

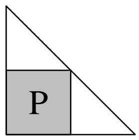
15. Eight teams participated in a football tournament, and each team played exactly once against each other team. If a match was drawn then both teams received 1 point; if not then the winner of the match was awarded 3 points and the loser received no points. At the end of the tournament the total number of points gained by all the teams was 61. What is the maximum number of points that the tournament's winning team could have obtained?

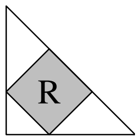
A 16 B 17 C 18 D 19 E 21

SOLUTION **B**

Let D be the number of drawn matches; then the number of points awarded for draws is $2D$ (one point for each team in the match). Let W be the number of matches that resulted in a win; then the number of points awarded for these matches is $3W$. Thus the total number of points is $2D + 3W = 61$ [1]. Each of the 8 teams played 7 others, so the number of matches is $8 \times 7 \div 2 = 28$ (each match has been counted twice, so we need to divide by 2). Since each match is either a draw or a win (for one team), we have $D + W = 28$ [2].
Equation [1] - 2 × equation [2] gives $W = 5$. So exactly 5 matches were won. Hence the maximum number of points that the winning team could have obtained is $5 \times 3 + 2 = 17$ (5 wins and 2 draws).

16. Two congruent isosceles right-angled triangles each have squares inscribed in them as shown. The square P has an area of $45\,\text{cm}^2$.
What is the area, in cm^2, of the square R?

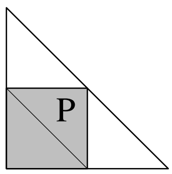


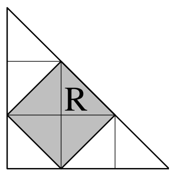


A 40 B 42 C 45 D 48 E 50

SOLUTION **A**

The diagram on the left shows a dissection of the triangle and square P into 4 congruent triangles. They are congruent because they each have the same angles (90°, 45°, 45°) and have one side whose length is equal to the side of the square. Since P has area $45\,\text{cm}^2$, each of the small triangles has area $22.5\,\text{cm}^2$. Hence the large triangle has area $90\,\text{cm}^2$.





The diagram on the right shows a dissection of the triangle and square R into 9 congruent triangles. They are congruent because they each have the same angles (90, 45, 45) and have one side whose length is equal to half a diagonal of the square. Each of these nine triangles has area, in cm^2, of $90 \div 9 = 10\,\text{cm}^2$. Hence the area of square R is $4 \times 10 = 40\,\text{cm}^2$.

17. Veronica put on five rings: one on her little finger, one on her middle finger and three on her ring finger. In how many different orders can she take them all off one by one?

A 16 B 20 C 24 D 30 E 45

Solution **B**

There are five options for when the ring on the little finger is removed. There are then four options for when the ring on the middle finger is removed. There are then no options for when the three rings on the ring finger are removed since they must be taken off in order in the three remaining slots. So there are $5 \times 4 = 20$ possible orders.

18. A certain city has two types of people: the 'positives', who only ask questions for which the correct answer is "yes" and the 'negatives' who only ask questions for which the correct answer is "no". When Mo and Bo met Jo, Mo asked, "Are Bo and I both negative?" What can be deduced about Mo and Bo?

A Both positive B Both negative
C Mo negative, Bo positive D Mo positive, Bo negative
E impossible to determine

Solution **C**

If Mo is positive, then the answer to his question must be "Yes" and that means he is negative, a contradiction. Hence Mo is negative. Therefore the answer to his question must be "No". So Mo and Bo cannot both be negative, and therefore Bo must be positive.

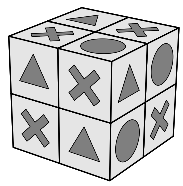
19. A group of pirates (raiders, sailors and cabin boys) divided 200 gold and 600 silver coins between them. Each raider received 5 gold and 10 silver coins. Each sailor received 3 gold and 8 silver coins. Each cabin boy received 1 gold and 6 silver coins. How many pirates were there altogether?

A 50 B 60 C 72 D 80 E 90

Solution **D**

Let R be the number of raiders, S the number of sailors and C the number of cabin boys. Then the number of gold coins is $5R + 3S + C = 200$ [1]. The number of silver coins is $10R+8S+6C = 600$ [2]. Subtracting [1] from [2] gives $5R+5S+5C = 400$, so $R+S+C = 80$.

20. Cuthbert is going to make a cube with each face divided into four squares. Each square must have one shape drawn on it; either a cross, a triangle or a circle. Squares that share an edge must have different shapes on them. One possible cube is shown in the diagram. Which of the following combinations of crosses and triangles is possible on such a cube (with the other shapes being circles)?



A 6 crosses, 8 triangles B 7 crosses, 8 triangles C 5 crosses, 8 triangles
D 7 crosses, 7 triangles E none of these are possible

Solution **E**

Each vertex of the cube consists of three squares each sharing a common edge with the other two. Hence each vertex must have one of each shape drawn on its three squares. Since there are 8 vertices, there must be 8 of each shape. Hence none of the options listed is possible.

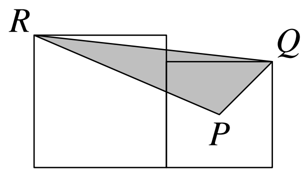
21. A grocer has twelve weights, weighing 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12 kilograms respectively. He splits them into three groups of four weights each. The total weights of the first and second groups are 41 kg and 26 kg respectively. Which of the following weights is in the same group as the 9 kg weight?

A 3 kg B 5 kg C 7 kg D 8 kg E 10 kg

Solution **C**

The total weight is $1 + 2 + \ldots + 12$ kg = 78 kg. The first two groups weigh 41 kg and 26 kg, leaving $(78 - 41 - 26)$ kg = 11 kg for the third group. Since this is only 1 kg heavier than the smallest possible combination of $(1 + 2 + 3 + 4)$ kg = 10 kg, there is only one way to combine 4 weights to get 11 kg, namely $(1 + 2 + 3 + 5)$ kg. The next smallest combination would then be $(4 + 6 + 7 + 8)$ kg = 25 kg, so the only way to get 26 kg would be $(4 + 6 + 7 + 9)$ kg = 26 kg. Hence the 9 kg weight is in the same group as the 7 kg.

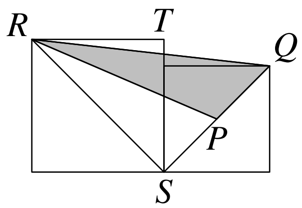
22. The bases of the two touching squares shown lie on the same straight line. The lengths of the diagonals of the larger square and the smaller square are 10 cm and 8 cm respectively. P is the centre of the smaller square. What is the area, in cm^2, of the shaded triangle PQR?



A 18 B 20 C 22 D 24 E 26

Solution **B**

Angles RST and TSQ are each 45° so triangle RSQ is a right-angled triangle with area, in cm^2, equal to $\frac{1}{2} \times RS \times QS = \frac{1}{2} \times 10 \times 8 = 40$. P is the midpoint of QS so the area of triangle PQR is half of the area of triangle RSQ, that is 20 cm^2.



23. The product of the digits of the positive integer N is 20.
One of the following could *not* be the product of the digits of $N + 1$. Which is it?

A 24 B 25 C 30 D 35 E 40

Solution **D**

There are two ways to make a list of digits whose product is 20: either use 2, 2, 5 and any number of 1s; or use 4, 5 and any number of 1s. Either way, none of the digits of N is 9 so the digits of $N + 1$ will be the same as the digits of N but with one of them increased by 1. Using 2, 2, 5 and any number of 1s gives the following possibilities: 3, 2, 5 and any number of 1s, with product 30; 2, 2, 6 and any number of 1s, with product 24; 2, 2, 5, 2 and any number of 1s, with product 40. Using 4, 5 and any number of 1s gives these possibilities: 5, 5 and any number of 1s, product 25; 4, 6 and any number of 1s, product 24; 4, 5, 2 and any number of 1s, product 40. The only option given that cannot be made is 35.

24. The lengths of the sides of pentagon $ABCDE$ are as follows: $AB = 16\,\text{cm}$, $BC = 14\,\text{cm}$, $CD = 17\,\text{cm}$, $DE = 13\,\text{cm}$, $AE = 14\,\text{cm}$. Five circles with centres at the points A, B, C, D, E are drawn so that each circle touches both of its immediate neighbours. Which point is the centre of the largest circle?

A B C D E

SOLUTION **A**

Let R_A, R_B, R_C, R_D, R_E be the radii, in cm, of the circles with centres at A, B, C, D, E respectively. Each side of the pentagon is equal to the sum of the radii of the circles whose centres are at its endpoints. That is:

$$AB = R_A + R_B = 16 \quad [1]$$
$$BC = R_B + R_C = 14 \quad [2]$$
$$CD = R_C + R_D = 17 \quad [3]$$
$$DE = R_D + R_E = 13 \quad [4]$$
$$EA = R_E + R_A = 14. \quad [5]$$

Adding these gives

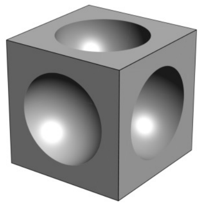
$$2(R_A + R_B + R_c + R_D + R_E) = 74$$
$$\text{so} \quad R_A + R_B + R_c + R_D + R_E = 37. \quad [6]$$

Adding [1] and [3] gives

$$R_A + R_B + R_c + R_D = 33. \quad [7]$$

Subtracting [7] from [6] gives $R_E = 4$.
Substituting R_E into [5] gives $R_A = 10$.
Substituting R_A into [1] gives $R_B = 6$.
Substituting R_B into [2] gives $R_C = 8$.
Substituting R_C into [3] gives $R_D = 9$. Hence the largest radius is R_A.

25. The cube shown has sides of length 2 units. Holes in the shape of a hemisphere are carved into each face of the cube. The six hemispheres are identical and their centres are at the centres of the faces of the cube. The holes are just large enough to touch the hole on each neighbouring face. What is the diameter of each hole?



A 1 B $\sqrt{2}$ C $2 - \sqrt{2}$ D $3 - \sqrt{2}$ E $3 - \sqrt{3}$

Solution **B**

Let P and Q be the centres of two adjacent hemispheres. The faces on which these hemispheres are carved meet at an edge. Let M be the midpoint of that edge. Then $MP = MQ = 1$. Also MPQ is a right-angled triangle since the two faces are perpendicular. By Pythagoras, $MP^2 + MQ^2 = PQ^2$, so $PQ^2 = 1 + 1 = 2$. Hence $PQ = \sqrt{2}$, and PQ is equal to the sum of two radii, so is the same as the diameter of the hemispheres.